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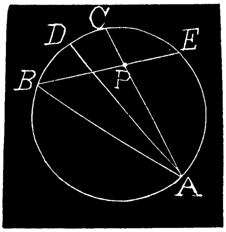
**32.** Proposed by F. P. MATZ, Sc. D., Ph. D., Professor of Mathematics and Astronomy in Irving College, Mechanicsburg, Pennsylvania.

Find the average area of the random sector whose vertex is a random point in a given circle.

**Solution by G. B. M. ZERR, A. M., Ph. D., Texarkana, Arkansas-Texas.**

Let $P$ be the random point. Through $P$ draw the chords $AC$, $DE$ forming the sector $DPC$. From $A$ draw the diameter $AB$ and the chord $AD$. Let $AB=2r$, $AP=z$, $\angle BAP=\varphi$, $\angle BAD=\theta$, area $DPC=u$, $\Delta=$required average. Then




$$u=r^2(\varphi-\theta-\tfrac{1}{2}\sin2\theta+\tfrac{1}{2}\sin2\varphi)-rz\cos\theta\sin(\varphi-\theta).$$

The limits of $\theta$ are $-\frac{1}{2}\pi$ and $+\frac{1}{2}\pi$; of $\varphi$, $\theta$ and $\frac{1}{2}\pi$; of $z$, 0 and $2r\cos\varphi=z'$.

$$\therefore\ \Delta=\frac{\int_{-\frac{1}{2}\pi}^{+\frac{1}{2}\pi}\int_{\theta}^{\frac{1}{2}\pi}\int_{0}^{z'}ud\theta d\varphi zdz}{\int_{-\frac{1}{2}\pi}^{+\frac{1}{2}\pi}\int_{\theta}^{\frac{1}{2}\pi}\int_{0}^{z'}d\theta d\varphi zdz}=\frac{2}{\pi^2r^2}\int_{-\frac{1}{2}\pi}^{+\frac{1}{2}\pi}\int_{\theta}^{\frac{1}{2}\pi}\int_{0}^{z'}ud\theta d\varphi zdz,$$

$$=\frac{2r^2}{3\pi^2}\int_{-\frac{1}{2}\pi}^{+\frac{1}{2}\pi}\int_{\theta}^{\frac{1}{2}\pi}[3\cos^2\varphi(2\varphi-2\theta-\sin2\theta+\sin2\varphi)-8\cos\theta\cos^3\varphi\sin(\varphi-\theta)]d\theta d\varphi$$

$$=\frac{r^2}{12\pi^2}\int_{-\frac{1}{2}\pi}^{+\frac{1}{2}\pi}(3\pi^2-12\pi\theta+\theta^2-16\cos^2\theta+4\sin^2\theta\cos^2\theta)d\theta=\frac{37\pi r^2}{144}-\frac{5r^2}{8\pi}.$$

Also solved by the *PROPOSER*.

**33.** Proposed by F. P. MATZ, Sc. D., Ph. D., Professor of Mathematics and Astronomy in Irving College, Mechanicsburg, Pennsylvania.

Find the average area of all regular polygons having a constant apothem.

**Solution by G. B. M. ZERR, A. M., Ph. D., Texarkana, Arkansas-Texas.**

Let $a=$constant apothem, $2x=$side, $2\theta=$central angle of polygon.

$\therefore\ \frac{\pi}{\theta}=$number of sides, $\frac{\pi ax}{\theta}=$area of polygon.

$$\therefore\ A=\text{average area}=\pi a\frac{\int_{0}^{a\sqrt{3}}\frac{xdx}{\theta}}{\int_{0}^{a\sqrt{3}}dx}=\frac{\pi}{\sqrt{3}}\int_{0}^{a\sqrt{3}}\frac{xdx}{\theta}$$

$$=\frac{\pi a^2}{\sqrt{3}}\int_{0}^{\frac{1}{3}\pi}\frac{\tan\theta\sec^2\theta d\theta}{\theta}\quad\text{where } x=a\tan\theta$$

$$=\frac{3\sqrt{3}a^2}{2}+\frac{\pi a^2}{2\sqrt{3}}\int_0^{\frac{1}{3}\pi}\left(\frac{\tan\theta}{\theta}\right)^2 d\theta$$

$$=\frac{3\sqrt{3}a^2}{2}+\frac{\pi a^2}{2\sqrt{3}}\int_0^{\frac{1}{3}\pi}(1+\tfrac{2}{3}\theta^2+\tfrac{17}{45}\theta^4+\tfrac{62}{315}\theta^6+\ldots\ldots)d\theta,$$

$$=\frac{3\sqrt{3}a^2}{2}+\frac{\pi^2a^2}{6\sqrt{3}}\left(1+\frac{2\pi^2}{81}+\frac{17\pi^4}{18225}+\frac{62\pi^6}{1607445}+\ldots\ldots\right)=3.8693a^2 \text{ nearly.}$$

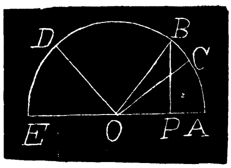
Also solved by the *PROPOSER*.

**34. Proposed by B. F. FINKEL, A. M., Professor of Mathematics and Physics in Drury College, Springfield, Missouri.**

Two points are taken at random on the circumference of a semicircle. Find the chance that their ordinates fall on either side of a point taken at randon on the diameter.

**Solution by G. B. M. ZERR, A. M., Ph. D., Texarkana, Arkansas-Texas.**

Let $P$ be the random point on the diameter $AE$. Draw $BP$ perpendicular to $AE$. Then one point must fall somewhere, as at $C$, on arc $AB$, the other somewhere, as at $D$, on arc $BE$. The chance thus obtained must be doubled as $D$ might fall on $AB$ and $C$ on $BE$.




Let $AO=$unity, $\angle BOA=\theta$, $\angle COA=\varphi$, $\angle DOA=\psi$.

Then $OP=\cos\theta$. $\therefore d(OP)=-\sin\theta d\theta$.

Let $p=$required chance.

$$\text{Then } p=\frac{\int_0^\pi\int_0^\theta\int_\theta^\pi \sin\theta d\theta d\varphi d\psi}{\int_0^\pi\int_0^\pi\int_0^\pi \sin\theta d\theta d\varphi d\psi}=\frac{1}{\pi^2}\int_0^\pi\int_0^\theta\int_\theta^\pi \sin\theta d\theta d\varphi d\psi$$

$$=\frac{1}{\pi^2}\int_0^\pi(\pi\theta-\theta^2)\sin\theta d\theta=\frac{4}{\pi^2}.$$

## PROBLEMS.

**42. Proposed by CHARLES E. MYERS, Canton, Ohio.**

A attends church 4 Sundays out of 5; B, 5 Sundays out of 6; and C, 6 Sundays out of 7. What is the probability of an event that A and B will be at church and C will not?

**43. Proposed by HENRY HEATON, M. Sc., Atlantic, Iowa.**

In a circle whose radius is $a$, chords are drawn through a point distant $b$ from the center. What is the average length of such chords, (1), if a chord is drawn from every point of the circumference, and (2), if they are drawn through the point at equal angular intervals?